Evaluation of Grounding Resistance and Inversion Method to Estimate Soil Electrical Grounding Parameters

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ABSTRACT

Soil resistivity plays a key role in designing grounding systems for high-voltage transmission lines and substations. The objectives of this paper are to determine the best estimated value of the apparent resistivity or electrode grounding resistance of N-layer soil and to use a new inversion method to precisely determine earth parameters. The inversion of electrical sounding data does not yield a unique solution, and a single model to interpret the observations is sought. This paper presents a new inversion method to statistically estimate soil parameters from Schlumberger and Wenner measurements. To validate the method and test the inversion scheme, four soundings were selected: two theoretical and two in the field. The procedure was applied using test data and a satisfactory soil model was obtained.

Index Terms: electrical grounding parameters, N-layer soil, apparent resistivity, resistivity measurement interpretation

I. INTRODUCTION

Ground electrode resistance plays an important role in designing power substations and transmission towers. Predicting this resistance is very complicated in some situations, especially when the soil is represented by an N-layer horizontal model with different resistivities.

Soil conductivity can vary across seasons and the depth of each layer is usually unknown, which makes determining the grounding resistance of an electrode particularly difficult.

The N-layer soil model is a realistic representation of the actual soil. Soil parameters can be estimated based on a set of soil resistivity measurements.

Several authors have contributed to solving the problem of soil parameter estimation to achieve the best fit between measured and computed resistivity values.

The Soil Measurements Interpretation Program (SOMIP) developed by Meliopoulos and Papalexopoulos [1] statistically estimates soil parameters based on resistivity measurements, but applied to field tests using four pin or three pin measurements. The program is designed to reject erroneous measurements resulting from instrument inaccuracies and changes in local soil resistivities.

Dawalibi and Barbeito [2] used imaging methods to compute the resistivity of multilayer soils by subdividing the soil into layers with thicknesses equal to a multiple of a base value.

The authors also used a method based on convolution and multiple filter theories to examine soil resistivity measurements in multilayer soil.

Takahashi and Kawase [3] developed a very compact formula to analyze changes in calculated apparent resistivity in multilayer soils, using a comparison of ρ-a curves. However, they did not provide an analytical solution to the problem of parameter estimation in such soils.

Del Alamo [4] compared eight different techniques to develop an optimum estimation of electrical grounding parameters for a two-layered soil model.

Lagacé et al. [5] used combined electrostatic images to evaluate N-layer soil resistivities and interpret sounding measurements from Wenner and Schlumberger electrode configurations.

Slaoui et al. [6] applied linear electric filter theory to derive the resistivity transform and proposed a method to estimate N-layer soil parameters from Schlumberger measurements using a ridge regression estimator.

Slaoui et al. [7] developed an innovative method to calculate the apparent resistivity of horizontally multilayered soils using an inversion method to determine the electrical grounding parameters of N-layered earth (resistivities and thicknesses).

Lagace and Vuong [8] proposed a method with a graphical user interface to estimate soil parameters of multilayered horizontal soil. The N-layer soil model illustrated in Figure 1 is a simplification of the actual soil.

This paper attempts to better estimate soil parameters from resistivity measurements. A statistical method to estimate the parameters of an N-layer soil model from Schlumberger measurements is presented.

This interpretation method works well for field data. Two field cases are presented. The two-sounding method was selected, not because the analysis was easy using the ridge regression method, but because the interpretation was very difficult using any method.
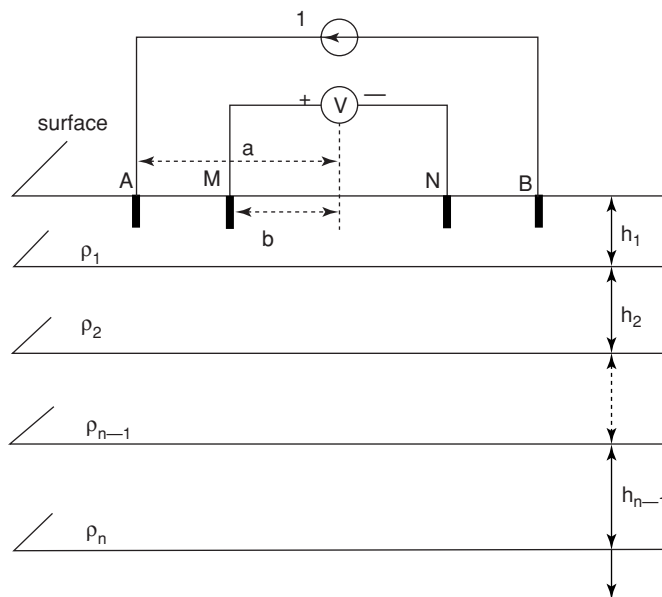

Figure 1   N-layer earth model with Schlumberger configuration.

II. APPARENT RESISTIVITY EXPRESSIONS

Resistivity data is conventionally expressed in terms of horizontal layers having an isotropic resistivity. A model consisting of n layers would be parameterized by a set of layer resistivities ρ_1, ρ_2, ..., ρ_n, and thicknesses h_1, h_2, ..., h_{n-1}. The nth layer is the deepest and extends to infinite depth. Reducing Slaoui's [7] calculation of theoretical apparent resistivity curves, the following general recurrence expression is obtained: (j = n − 1, n − 2, ..., 2, 1).

USING SCHLUMBERGER'S ARRANGEMENT

$$\rho_{jas}(a,b) = \rho_j \left[1 + \sum_{i=1}^{\infty} k_j^i G(a,b,h_j,i) \right] \tag{1}$$

$$k_j = \frac{\rho_{(j+1)aS} - \rho_j}{\rho_{(j+1)aS} + \rho_j} \tag{2}$$

$$\rho_{naS} = \rho_n \tag{3}$$

$$G(a,b,h_j,i) = U \left[\frac{1}{\sqrt{(2h_j i)^2 + (a-b)^2}} - \frac{1}{\sqrt{(2h_j i)^2 + (a+b)^2}} \right] \tag{4}$$

where $U = \dfrac{a^2 - b^2}{b}$

ρ_j is the resistivity of the jth layer,
h_j is the thickness of the jth layer,
ρ_{jaS} is the Schlumberger apparent resistivity at the jth layer,
n is the number of layers in the soil model,
a is half the distance between current electrodes, and
b is half the distance between voltage electrodes.
The apparent resistivity ρ_{aS} of the N-layer model is obtained at j = 1.

III. INVERSION METHOD WITH RIDGE REGRESSION

The problem of Schlumberger sounding over a plane-layered earth is nonlinear in the unknown parameters, namely, the resistivity and thickness of each layer.

As a starting point, we have a series i (i = 1, 2,..., N) of resistivity measurements ρ_{mS}, and N is the number of measurements.

In order to estimate soil parameters in multilayer soils from resistivity measurements, the least squares fitting error between the measured values must be minimized and apparent values must be analytically computed for the same spacing.

The objective function to minimize is

$$F = \sum_{i=1}^{N} (\rho_{iaS} - \rho_{imS})^2 \tag{5}$$

LEVENBERG-MARQUARDT METHOD

The Levenberg-Marquardt method is used to minimize multivariate functions. In N-layer soil, the values $X = (\rho_1, \rho_2, ... \rho_n, h_1, h_2, ... h_{n-1})^T$ must be determined in order to minimize the distance function F. The F gradient is equal to zero at the absolute minimum. The Levenberg-Marquardt method consists of iteratively computing the F gradient by adjusting X from the Jacobien J such that it reduces the values of the gradient:

From [4, 9], the gradient of F can be evaluated at $X + \Delta X$:

$$\nabla F(X + \Delta X) = 2J^T(\rho_{aS}(X + \Delta X) - \Delta_{mS}) \tag{6}$$

where

$$\rho_{aS} = (\rho_{1aS}, \rho_{2aS}, ..., \rho_{NaS})^T \tag{7}$$

$$\rho_{mS} = (\rho_{1mS}, \rho_{2mS}, ..., \rho_{NmS})^T \tag{8}$$

$$J_{i,j} = \frac{\partial \rho_{iaS}}{\partial X_j} \quad i = 1, ..., N \text{ and } j = 1, ..., (2n-1) \tag{9}$$

where J is a Jacobien matrix with partial derivatives of weighted differences with respect to each parameter. The apparent resistivity can be extended in the vicinity of X:

$$\rho_{aS}(X + \Delta X) = \rho_{aS}(X) + J\Delta X \tag{10}$$

Equation (10) represents a linear system of N equations with M unknowns, where the number of unknowns is $2n - 1$. Since this is a nonlinear problem, by substituting (10) in (6), the application of the least-squares inverse method yields

$$J^T J \Delta X + J^T(\rho_{aS}(X) - \rho_{mS}) = 0$$
$$\Delta X = (J^T J)^{-1} J^T(\rho_{aS}(X) - \rho_{mS}) \tag{11}$$

The method begins by calculating the eigenvalues of the matrix $J^T J$. Small eigenvalues indicate a near-singular system, which is unstable in the presence of noisy data, and the average difference between estimated ΔX and true ΔX becomes very large. The ridge regression method [9,11] seeks to reduce this difference during the iteration process by damping the diagonal terms of $(J^T J)$.

The ridge regression estimate of ΔX_{rr} is

$$\Delta X_{rr} = (J^T J + \alpha I)^{-1} J^T(\rho_{aS}(X) - \rho_{mS}) \tag{12}$$

where I is the unit identity matrix and $\alpha \geq 0$.

In the above equation, $\alpha = 0$ and ΔX approaches the Taylor series direction. The eigenvalues of $(J^T J + \alpha I)^{-1}$ are $(\lambda_i^2 + \alpha)$, whereas λ_i^2 are the eigenvalues of $J^T J$. Increasing the size of all eigenvalues results in significant decreases in a) the mean of the squared length between ΔX and ΔX_{rr} and b) the variance of the estimated solution. Therefore, in some cases, the solution ΔX_{rr} is much closer to ΔX than the standard least-squares solution. The residual sum of squares for the ridge regression solution is given by

$$\Delta S = \sum_{i=1}^{N}(\rho_{ias} - \rho_{ims})^2 = (\Delta \rho^*)^T \Delta \rho^* \tag{13}$$

where $\Delta \rho^* = \rho_{aS} - \rho_{mS}$ is using the values $X^* = X + \Delta X_{rr}$.

IV. DATA ERROR AND WEIGHTING

Generally, field curves contain noise due to measuring errors, superficial inhomogeneities, telluric noise, and limited instrumentation precision.

When data are weighted, a relative degree of importance is assigned to each value. Such weighting may be used to remove inherent bias in the data or to bias the least squares fit so that it is more accurate in one area of the curve than another.

If there is a large numerical difference between the data values in different regions of the curve, an undesirable bias may be introduced into the final solution. The bias is such as to

cause the ridge regression estimator to be biased toward the large values while neglecting the smaller values, which may be as accurate and may contain some very important information.

In general, it is desirable to weight each data point according to the noise in that data point, and so as not to give it a false degree of importance due to its large or small value compared with other data points.

We assume that each data point has the same percentage standard deviation unless it is known or suspected that certain points are significantly noisier. It is further assumed, initially, that each point has a standard deviation equal to one percent of its measured value. The problem standard deviation σ is then adjusted to the estimated noise level of the survey. Most resistivity surveys yield data that is accurate within five per cent or less.

The solution consists of incorporating the weights within the residual sum of squares for the ridge regression, as given by

$$\Delta S = \sum_{i=1}^{N} \left(\frac{\rho_{iaS} - \rho_{imS}}{\sigma_i} \right)^2 \tag{14}$$

The residual error is now defined as

$$\varphi = (\Delta\rho - J\Delta X)^T W^{-1} (\Delta\rho - J\Delta X) \tag{15}$$

If the error in each measurement is independent of the error in other measurements, as is usually assumed, then W reduces to a diagonal matrix of data variance σ_i^2:

$$W_{ij} = \begin{cases} \sigma_i^2 & i = j \\ 0 & i \neq j \end{cases} \tag{16}$$

The choice of a weighting scheme does not appreciably affect minimization speed. However, it can drastically affect the position of the minimum in parameter space and the parameter statistics.

The weighted ridge regression estimator is given as

$$\Delta X_{rr} = (J^T W^{-1} J + \alpha I)^{-1} J^T W^{-1} \Delta\rho \tag{17}$$

V. PARAMETER STATISTICS
The next most important requirement is to obtain some idea about parameter uncertainty. The statistical parameters for our models are a) parameter standard errors and b) the parameter correlation coefficient. In addition to these statistical parameters, the parameter and data eigenvectors with the associated eigenvalues can yield great insight into the relations between individual model parameters and specific data.

Parameter standard errors and correlations are derived from the covariance matrix cov(X) evaluated at the minimum, as shown in [1, 11]:

$$\text{cov}(X) = \bar{\sigma}^2 (J^T W^{-1} J)^{-1} \tag{18}$$

$$\text{where } \bar{\sigma}^2 = \frac{\Delta\rho^T W^{-1} \Delta\rho}{N - M} \tag{19}$$

When $\bar{\sigma}^2$ is greater than σ^2, σ^2 has been underestimated. Although the value of σ^2 varies between surveys, it is assumed that the error at any data point is five percent or less. If $\bar{\sigma}^2$ is found to be smaller than that of the weighting, either the estimated variance of the observations have been overestimated or the curve calculated from the hypothesized model fits the noise in the data.

Thus, the residual variance can be used as an indicator of goodness-of-fit. The residual variance is independent of the linearity or nonlinearity of the problem with respect to the model parameters.

The parameter standard errors are defined by the square root of the diagonal term of $cov(X)$ (e.g., $[cov(X)_{ii}]^{1/2}$ equals the standard error for numbered parameter i).

The correlation matrix is an indicator of the linear dependence between parameters. The correlation matrix elements are given in [2, 11].

$$[cor(X)]_{ij} = \frac{[cov(X)]_{ij}}{[cov(X)]_{ii}^{1/2}[cov(X)]_{jj}^{1/2}} \tag{20}$$

If the value of $[cor(X)]_{ij}$ is near unity, then the parameters X_i and X_j are strongly correlated and nearly linearly dependent. For example, if i represents the thickness h and j, the resistivity ρ of a layer (i.e., $[cor(X)]_{ij}$ represents the correlation between the thickness and resistivity of a layer), only the ratio h/ρ is well determined by the data if $[cor(X)]_{ij} = 1$. This is true for layers that are highly conductive relative to their surroundings. If $[cor(X)]_{ij} = -1$, only the product $h\rho$ is well determined, as is the case for relatively resistive layers. This is the familiar equivalence problem, as discussed by Sunde [10].

The diagonal elements of the covariance matrix are the variance terms for each parameter. If the correlations are small, then the standard deviation is a good measure of the uncertainty of each parameter. If two parameters are strongly correlated, then the standard deviation given by the square roots of the diagonal terms of (18) will be larger than the actual uncertainties.

The eigenvectors and their associated eigenvalues are also very useful in defining the relations between soil parameters and their effects on data generated from a particular model.

The generalized inverse of J is defined in terms of these eigenvectors and eigenvalues, as described by Sunde [10].

$$J^{-1} = BA^{-1}C^{T} \tag{21}$$

The matrix C consists of r (r is a rank of J) eigenvectors C_i of length N associated with the columns (data) of J. B comprises the r eigenvectors B_i of length M associated with the rows (parameters) of J. The matrix A^{-1} is the inverse of the diagonal matrix comprised of the eigenvalues of J.

VI. EXAMPLES OF DATA INTERPRETATION

In this section, four Schlumberger sounding curves and their associated models illustrate the ridge regression method and the estimation method for all parameter statistics.
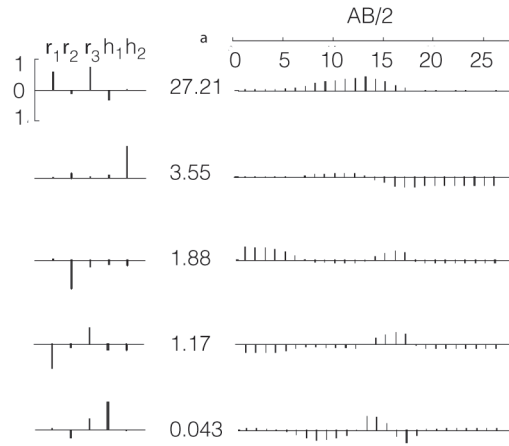
A. THEORETICAL EXAMPLES

Consider theoretical model #1: X=(15, 500, 50, 10, 150), with a resistive middle layer.

In Figure 2, the first column shows the eigenvectors (columns of B), the second column shows the eigenvalues (diagonal elements of A), and the third column shows the data eigenvectors (columns of C), where the abscissas represents the number of measurements. The correlation coefficients and the model parameter values with estimated standard errors, calculated using data errors as weights in equation (14), are also shown.

The solid curve in Figure 3 is the final fit to the data points, which appears to be very good because there is little data noise. The final estimated model is close to the original model.

The first three eigenvalues have high magnitude. The linear combination of parameters represented by the first three eigenvectors has the greatest effect on the sounding curve. In

Correlation matrix					True model	Best fit	Standard error
	ρ_1	ρ_2	ρ_3	h_1	h_2			
ρ_1	1					15	15	±0.0116
ρ_2	0.09	1				500	500	±0.5424
ρ_3	0.01	0.16	1			50	50	±0.0080
h_1	0.94	0.39	0.05	1		10	10	±0.0085
h_2	−0.23	−0.91	−0.32	−0.47	1	150	150	±0.0474

Figure 2 Parameters and data eigenvectors with associated eigenvalues, parameter correlations, and best fit model parameters for minimization with standard error weighted data.

these three eigenvectors, the elements ρ_1 and h_1 have opposite signs, whereas the elements ρ_2 and h_2 have the same sign. This indicates that if ρ_2 and h_2 are both changed in the same direction, the effect on the sounding curve (Figure 3) will be larger than the effect of similar
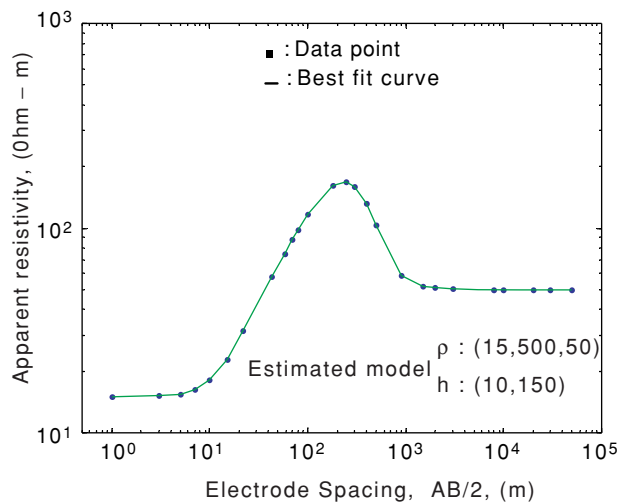

Figure 3   Theoretical data and best fit curve for three-layer resistivity model #1 with Schlumberger configuration.

changes on other parameters. In addition, if ρ_1 increases while h_1 decreases, or vice versa, the sounding curve will change accordingly.

The eigenvector associated with $\lambda_4 = 1.17$ indicates that increasing or decreasing ρ_1 and h_1 together will have little effect on the sounding curve (Figure 3). In other words, the ratio h_1/ρ_1 is the combination of these parameters that affects the sounding curve the most (note that the correlation coefficient between ρ_1 and h_1 is $+0.94$). The eigenvector associated with $\lambda_5 = 0.043$ indicates that increasing ρ_2 while decreasing h_2, or vice versa, has little effect on the sounding curve. Again, this is also indicated by considering the correlation coefficient between ρ_2 and h_2, which is -0.91.

Considering the parameter eigenvector and parameter correlation coefficients of this model, it can be seen that the products $\rho_2 h_2$ are better determined by, and have greater effect on, this data than either ρ_2 or h_2 separately.

From the analysis of the eigenvectors, we see a wide range of resistivities and thicknesses for a very resistive layer that will yield nearly identical sounding curves as long as the thickness-resistivity ratio changes only slightly. In addition, for a very conductive layer, there is large range of resistivities and thicknesses that yield nearly the same sounding curve as long as the resistivity-thickness product is relatively constant. Thus, the parameters of highly resistive or highly conductive layers are difficult to determine. The correlation matrix must be considered in any interpretation of the standard deviations given by equation (18).

For the second example, we consider four-layer earth model #2: X(150, 700, 15, 200, 3, 20, 40). Figure 4 shows the theoretical data points for this model. The solid curve in Figure 4 is the final fit to the data points, and the final estimated model is close to the original model.

Figure 5 illustrates the parameter eigenvectors, eigenvalues, data eigenvectors, parameter correlation coefficients, and model parameter values with estimated standard errors.

In this example, we note five large eigenvalues and two small eigenvalues.

Due to the several high eigenvalues, the inverse interpretation for this model is not very sensitive to noise, but it is very sensitive to model parameter changes. The eigenvectors may
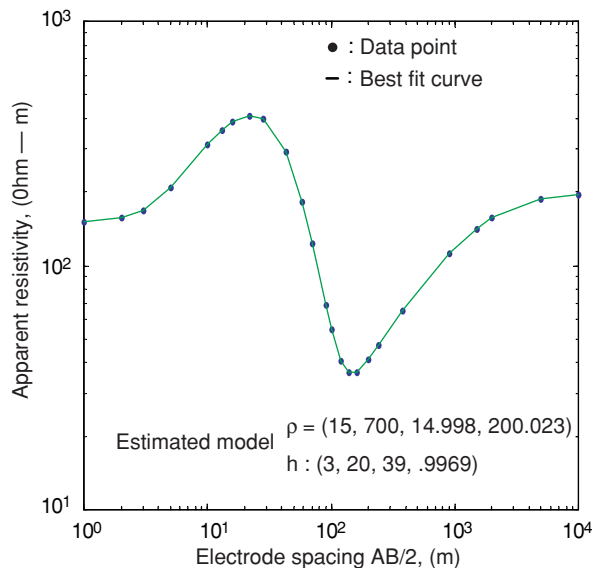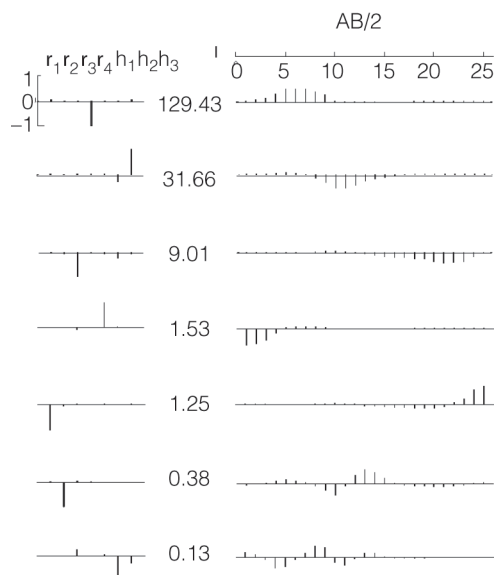

Figure 4   Theoretical data and best fit curve for a four-layer resistivity model #2 with Schlumberger configuration.

Correlation matrix							True model	Best fit	Standard error
	ρ_1	ρ_2	ρ_3	ρ_4	h_1	h_2	h_3			
ρ_1	1							150	150	±0.0013
ρ_2	0.30	1						700	700	±0.0138
ρ_3	−0.13	−0.56	1					15	14.99	±0.0015
ρ_4	−0.003	−0.01	−0.14	1				200	200.02	±0.0015
h_1	0.71	0.84	−0.41	−0.01	1			3	3	±0.0001
h_2	−0.40	−0.89	0.27	−0.05	−0.82	1		20	20	±0.0002
h_3	−0.14	−0.58	0.98	−0.06	−0.42	0.3	1	40	38.99	±0.0062

Figure 5 Parameters and data eigenvectors with associated eigenvalues, parameter correlations, and best fit model parameters for minimization with standard error weighted data.

be interpreted as for the three-layer problem. The eigenvectors associated with the small eigenvalues indicate the expected combination of parameters to which the sounding curve is insensitive, and the eigenvectors associated with the large eigenvalues indicate the combinations of parameters to which the problem is highly sensitive. For example, the eigenvector of $\lambda_6 = 0.38$ indicates that if the ρ_2 value increases while h_2 decreases, or vice versa, there will be almost no effect on the sounding curve.

B. FIELD EXAMPLE
The field data were taken from the Bryson site by Hydro-Québec. Consider the interpretation of the three-layer earth model #3. Figure 6 contains the data for the Schlumberger sounding and its computed sounding curve. The curve fits the data quite well, and the estimated noise level based on this fit is approximately 4 percent of the value at several data points. Since this is considered the approximate accuracy of the data, a closer fit is not justified.

Table 1 illustrates the parameter correlation coefficients, and the model parameter value with estimated standard errors. All calculated parameters may be interpreted as for the three-layer
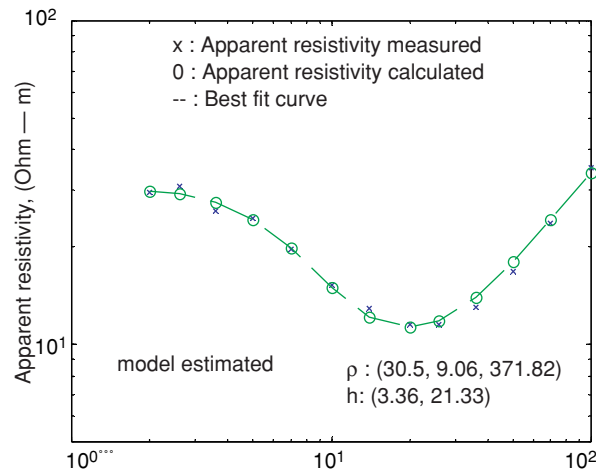
Figure 6   Field data from the Bryson site (Hydro-Québec) and interpreted best fit
curve for three-layer resistivity model #3.

problem in model #1. The data do not appear to provide much information about the third
layer: we expected a much higher standard deviation for resistivity ρ_3. Another interesting
observation is the strong correlation between parameter ρ_3 and the thickness of the second
layer.

Table 1   Parameter correlations and best fit model parameters for minimization with
standard error weighted data

Correlation matrix						Best fit	Standard error
	ρ_1	ρ_2	ρ_3	h_1	h_2		
ρ_1	1					30.52	±0.9457
ρ_2	0.36	1				9.06	±1.2779
ρ_3	0.11	0.46	1			371.82	±618.389
h_1	−0.64	−0.81	−0.31	1		3.36	±0.3653
h_2	−0.23	0.89	−0.79	−0.71	1	21.33	±5.8902

The final example, an interpretation of resistivity sounding measurements, was analyzed
for a four-layer case using the Schlumberger configuration in Table 2.

Figure 7 illustrates the measured and calculated apparent resistivities, showing better
correlation.

Table 3 illustrates the parameter eigenvectors, eigenvalues, and data eigenvectors.

Table 4 illustrates the parameter correlation coefficients and the model parameter value
with estimated standard errors.

All calculated parameters may be interpreted as for the four-layer problem in model #2.
The data do not appear to provide much information about the fourth layer: we expected a
higher standard deviation for the resistivity ρ_4. Another interesting observation is the strong
correlation between parameter ρ_3 and the thickness of the second and third layers.

Table 2   Numerical values of the 18 Schlumberger array measurements

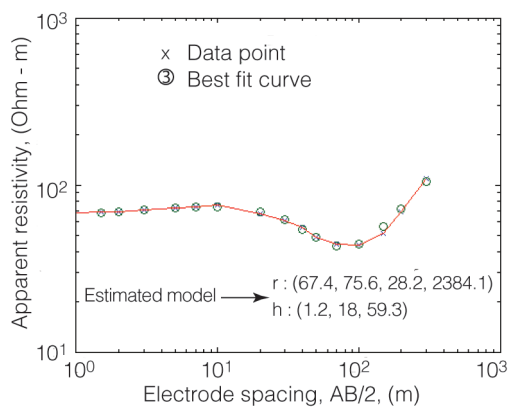| Electrode separation (m)	Apparent resistivity (Ω–m)
1	67.7
1.5	68.7
2	69.3
3	70.8
5	72.8
7	73.9
10	76.1
20	66.8
30	61.2
40	55.3
50	48.6
70	44.5
100	43.9
150	51.5
200	68.7
300	109.2

Figure 7   Field data and interpreted best fit curve for four-layer resistivity model #4.

Table 3   Parameter eigenvectors, eigenvalues, and data eigenvectors

Eigenvalues	Parameter eigenvectors						
	σ_1	σ_2	σ_3	σ_4	h_1	h_2	h_3
7.243	−0.153	0.038	−0.191	0.736	−0.628	0.015	−0.000
5.452	−0.075	−0.022	−0.474	−0.644	−0.592	−0.592	−0.000
3.280	−0.111	−0.941	−0.069	0.058	0.067	0.297	−0.0007
1.247	−0.000	−0.000	0.0001	−0.0001	−0.0002	−0.003	−1.0000
0.770	0.952	−0.134	0.121	0.028	−0.244	0.0111	−0.000
0.088	0.219	0.039	−0.845	0.192	0.428	−0.125	0.0002
0.0004	0.059	0.304	-0.064	0.027	0.079	0.944	−0.0034

Table 4   Parameter correlations and best fit model parameters for minimization with standard error weighted data

	ρ_1	ρ_2	ρ_3	ρ_4	h_1	h_2	h_3	Best fit	Standard error
ρ_1	1							67.4	±0.0029
ρ_2	0.3852	1						75.6	±0.0031
ρ_3	0.1525	0.460	1					28.2	±0.0107
ρ_4	0.0218	0.075	0.460	1				2384.1	±6.7673
h_1	0.7656	0.735	0.306	0.045	1			1.2	±0.0010
h_2	−0.3647	−0.7253	−0.902	−0.305	−0.606	1		18	±0.0046
h_3	0.1462	0.4414	0.9846	0.6028	0.2938	−0.875	1	59.3	±0.0378

VII. ELECTRODE RESISTANCE EXPRESSIONS

Using an N-layer earth model, as shown in Figure 8, the electrode resistance can be approximated using Wenner's arrangement.

The general recurrence expression for Wenner apparent resistivity is
(j = n − 1, n − 2, ..., 2, 1),

$$\rho_{jaW}(c) = \rho_j \left[1 + \sum_{i=1}^{\infty} k_j^i H(c, h_j, i) \right] \tag{22}$$

$$k_j = \frac{\rho_{(j+1)aW} - \rho_j}{\rho_{(j+1)aW} + \rho_j} \tag{23}$$

$$\rho_{na}W = \rho_n \tag{24}$$

$$H(c, h_j, i) = 4c \left[\frac{1}{\sqrt{(2h_j i)^2 + c^2}} - \frac{1}{\sqrt{(2h_j i)^2 + (2c)^2}} \right] \tag{25}$$

where

ρ_j is the resistivity of the jth layer,
h_j is the thickness of the jth layer,
ρ_{jaW} is the Wenner apparent resistivity at jth layer,
n is the number of layers in the soil model, and
C is the distance between any two electrodes.

The apparent resistivity ρ_{aw} of the N-layer model is obtained at j = 1:

$$R_e = \frac{\rho_{1aW}(r)}{2\pi r} \tag{26}$$

where

R_e is the ground electrode resistance Ω,
and r is the electrode radius m.
The height of the top layer (i.e., h_1) varies.
When h_1 increases, the approximation of the electrode resistance becomes more accurate.
When h_1 approaches infinity (homogeneous soil), the expression reduces to

$$\rho_{1aW}(r) \approx \rho_1$$

$$R_e = \frac{\rho_1}{2\pi r} \tag{27}$$

This is the expression for the resistance of hemisphere buried in a homogeneous soil.

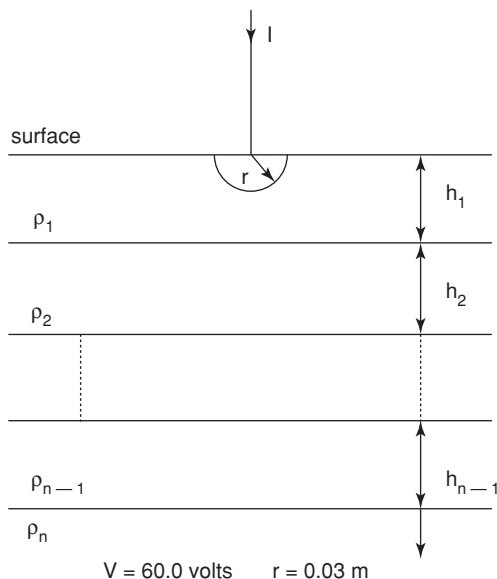The computed ground electrode resistance and power loss are tabulated in Table 5 for varying values of h_1.

Figure 8   Hemisphere buried in N- Layer soil.

Table 5   Electrode resistance and power loss vs. height

Three-layer soil with earth resistivity (Ω-m): ($\sigma_1 = 24$, $\sigma_2 = 100$, $\sigma_3 = 500$)

Height, h_1 (cm)	Electrode resistance using equation (26)	Electrode resistance using equation (27)	Power loss using equation (26)	Power loss using equation (27)
1.5	233.0345	127.3239	15.4483	28.2743
3	161.2195	127.3239	22.3298	28.2743
4.5	140.9845	127.3239	25.5347	28.2743
6	133.9004	127.3239	26.8856	28.2743
7.5	130.9250	127.3239	27.4966	28.2743
9	129.4894	127.3239	27.8015	28.2743
10.5	128.7206	127.3239	27.9675	28.2743
21	127.5079	127.3239	28.2335	28.2743
23	127.4645	127.3239	28.2431	28.2743
25	127.4337	127.3239	28.2499	28.2743
27	127.4112	127.3239	28.2549	28.2743
37	127.3582	127.3239	28.2667	28.2743
57	127.3334	127.3239	28.2722	28.2743
77	127.3258	127.3239	28.2739	28.2743
80	127.3242	127.3239	28.2743	28.2743

VIII. CONCLUSION

The ridge regression estimator is a powerful method for interpreting resistivity soundings over plane-layered earth structures. Accordingly, it is possible to find a model that fits the data, to indicate the accuracy of the fit relative to the noise level in the data, and to predict the accuracy with which each parameter is estimated.

The inversion method is based on a statistical estimation of the parameters of an N-layer soil model from Schlumberger measurements. This method provides the best estimate of soil parameters.

Finally, computation of the ground electrode resistance of hemisphere buried in N-layer soil is easier and faster using this method.

IX. APPENDIX

In order to obtain a recursive relationship for the partial derivatives, we differentiate Eq. (1):

$$\frac{\partial \rho_{1aS}}{\partial \rho_1} = 1 \quad \text{and} \quad \frac{\partial k_1}{\partial \rho_1} = \frac{-2\rho_2}{(\rho_1 + \rho_2)^2}$$

Using these expressions for j = 2, 3, ..., n, recursive application gives the partial derivatives of $\rho_{aS} = \rho_{naS}$

$$\frac{\partial k_{j-1}}{\partial \rho_j} = \frac{2\rho_{(j-1)aS}}{(\rho_j + \rho_{(j-1)aS})^2}$$

$$\frac{\partial k_{j-1}}{\partial \rho_{(j-1)aS}} = \frac{-2\rho_j}{(\rho_j + \rho_{(j-1)aS})^2}$$

Introducing $G_i(a, b, d_j, i) = G_i$, we have:

$$\frac{\partial \rho_{jaS}}{\partial \rho_j} = \rho_{(j-1)aS} \sum_{i=1}^{+\infty} i K_{j-1}^{i-1} G_i \frac{\partial K_{j-1}}{\partial \rho_j}$$

$$\frac{\partial G_i}{\partial h_{j-1}} = 4 U d_j i^2 \left(\frac{1}{B^3} - \frac{1}{A^3} \right)$$

$$A = \sqrt{(2d_j i)^2 + (a-b)^2}$$

$$B = \sqrt{(2d_j i)^2 + (a+b)^2}$$

$$\frac{\partial \rho_{jaS}}{\partial h_{j-1}} = \rho_{(j-1)aS} \sum_{i=1}^{+\infty} K_{j-1}^i \frac{\partial G_i}{\partial h_{j-1}}$$

For k = 1, 2, ..., j − 1, we have:

$$\frac{\partial k_{j-1}}{\partial \rho_k} = \frac{-2\rho_j \frac{\partial \rho_{(j-1)aS}}{\partial \rho_k}}{(\rho_j + \rho_{(j-1)aS})^2}$$

$$W = \sum_{i=1}^{+\infty} k_{j-1}^i G_i(a, b, d_j, i)$$

$$\frac{\partial W}{\partial \rho_k} = \sum_{i=1}^{+\infty} iK_{j-1}^{i-1} G_i \frac{\partial k_{j-1}}{\partial \rho_k}$$

$$\frac{\partial \rho_{jaS}}{\partial \rho_k} = \frac{\partial \rho_{(j-1)aS}}{\partial \rho_k} W + \rho_{(j-1)aS} \frac{\partial W}{\partial \rho_k}$$

$$\frac{\partial k_{j-1}}{\partial h_k} = \frac{-2\rho_j \dfrac{\partial \rho_{(j-1)aS}}{\partial h_k}}{(\rho_j + \rho_{(j-1)aS})^2}$$

$$Q = \sum_{i=1}^{+\infty} ik_{j-1}^{i-1}\left(G_i\left(\frac{\partial k_{j-1}}{\partial h_k}\right) + k_{j-1}^i \frac{\partial G_i}{\partial h_k}\right)$$

$$\frac{\partial \rho_{jaS}}{\partial h_k} = \frac{\partial \rho_{(j-1)aS}}{\partial h_k} \cdot W + \rho_{(j-1)aS} \cdot Q$$

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